MD Dialog on: Optimum Savings and Optimal Growth: the Cass-Malinvaud-Koopmans Nexus

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Introduction

In 1965, David Cass's "Optimum Growth in an Aggregative Model of Capital Accumulation" was published in *Review of Economic Studies* (*RES*). In the same year, Tjalling Koopmans's "On the Concept of Optimal Economic Growth" -- a Cowles Foundation Discussion Paper, originally "prepared for" (1963d,1), and presented at, a Vatican "Study Week" (1963c), 7-14 October 1963, on the subject of "The role of econometric analysis in economic development plans and in controlling econometric fluctuations" -- appeared in the Vatican conference volume (1965). Since then, the approach to optimum growth they developed -- "within the closed, aggregative model introduced by Solow (1956) and Swan(1956)" (Cass, 1965a, 1; 1965b, 233) , and based upon *significant* "elaboration," as Cass put it (1965a, 1; 1965b, 233) of Ramsey (1928), has been called by some, the "Ramsey-Cass-Koopmans" model. Others have described the one-sector optimum growth model as being "developed independently and simultaneously by Cass (1965) and Koopmans (1965)" (Lucas, Stokey, and Prescott, 1989, 35). Some have also asserted that "the Solow model was extended during the 1960's, especially by Cass (1965) and Koopmans (1965) and became known as the neoclassical growth model" (Barro, 2008, 53); while still others have termed the "Cass-Koopmans" approach the "general equilibrium version" of the "Solow-Swan" model (Durlauf and Quah, 1999).

Up to now, the development and dissemination of Cass's early work on optimum growth has *not* been documented, nor has the relationship between his work, and that of Koopmans. But the development of Cass's work, and the Cass-Koopmans connection, however, is not the only *untold* part of the origins of the optimum growth approach, for in the late 1940s, and early 1950s, Maurice Allais and Edmond

Malinvaud also initiated investigations into the problem of optimum capital accumulation and growth, with the work of Malinvaud having a significant impact on *both* the early work of Koopmans on the problem, and his later approach to growth, as will be shown here (Allais,1947, 1959, 1962; Malinvaud, 1951a,b,c,d; 1953, 1963,1965a,b; Koopmans, 1957, 1963c, 1965). In this paper, we will reconstruct the evolution of the Cass-Malinvaud-Koopmans nexus as the outcome of a process of *sequential cross-fertilization* of ideas, between Malinvaud and Koopmans, on the one hand, and Cass and Koopmans, via Uzawa, on the other, that occurred at three conferences over the period July-December 1963: the Cambridge UK Conference in July, the Vatican Conference in October, and the Econometric Society Conference in December respectively.

In the first section of this paper, we focus on the development and dissemination of Cass's one-sector approach to optimum growth -- based upon an analysis of the evolution of his work over the period 1963-65, in the context of his recollections (interview, 1998). We show how his *RES* paper (1965b) evolved from his Stanford thesis (1965a) and Chicago summer fellowship, and how his thesis work was influenced by his advisor, Hirofumi Uzawa. Cass's previously unknown *two-sector* model, and his turnpike model as published in *Econometrica* (1966), both of which were chapters in his Stanford thesis**,** will be dealt with in other forthcoming papers.

In the second section of this paper, we survey the contributions of Allais and Malinvaud, focusing on the work and presentations of Allais (1947, 1959, 1962), and Malinvaud's research at Cowles (1951a,b,c,d), manifest in a paper Malinvaud presented at the September 1951 Econometric Society meeting (1951c), and a Cowles Discussion Paper entitled "Capital Accumulation and Efficient Allocation of

Resources" (1951d), both of which were the basis for his seminal 1953 *Econometrica* paper, of the same title, that was later extensively analyzed by Koopmans (1957).

The third part of the paper focuses on Koopman's contributions to growth over the period 1957-67. This is done in order to provide the basis for an analysis and assessment -- in the fourth part of the paper -- of what we call the "Cass-Malinvaud-Koopman's nexus". In this part, we discuss evidence relating to the sequential cross-fertilization of ideas between Malinvaud and Koopmans, on the one hand, and Cass and Koopmans, via Uzawa, on the other, over the period July-December 1963, as stated above. We first deal with the previously overlooked *discussion* of papers presented at the July 1963 Cambridge UK Conference on "Activity Analysis in the Theory of Growth and Planning", especially with regard to the papers of Hurwicz, Radner, Chakravarty, and Koopmans' comments. It will be shown that one of the key things to come out of the discussion was the applicability and efficacy of Pontryagin (1962a,b) to problems in economic growth. We then deal with Malinvaud's critique of Koopman's *original* optimal growth approach in the course of the discussion of Malinvaud's paper at the Vatican conference *in October 1963*. The mathematics of the published version of Koopmans's Vatican paper (1965) is counterpointed to that in its original version (1963d; Cowles Foundation Discussion Paper 163, 6 December 1963), and the difference between them, regarding transversality, are discussed, in light of Malinvaud's "counter-example" pointing to an error in Koopman's approach. We then turn to the Dec. 1963 meeting of the Econometric Society, where Koopmans presented a version of his Oct. 1963 Vatican paper, which was a Cowles Foundation Discussion Paper dated 6 December (1963d) and its Appendix, dated 19 December (1963e) respectively. After a discussion of the conundrum in the original Appendix to Koopmans' paper, we turn to survey the sessions and participants at the Econometric

Society conference, at which Koopmans presented his optimal growth paper, so as to assess possible paths of cross-fertilization. In the final part of the paper, we sum up the evidence to the effect that the 1965 papers of Cass and Koopmans may actually be the outcome of sequential cross-fertilization between them, via Uzawa; and this, as against the view of most observers, including Lucas, Stokey and Prescott (1989), and Arrow (2009), who have *implicitly* assumed that the 1965 papers of Cass and Koopmans were "developed *independently* and *simultaneously*" [our emphasis].

Briefly put, then, we deal with the origins and evolution of Cass's 1965 *RES* paper on optimum growth, and those of Malinvaud and Koopmans in the Vatican Conference Volume (1965) by means of:

(i) *analyzing* the *thesis* chapter and published version of Cass (1965a,b) in the context of his recollections (Cass interview, 1998). The work of Allais (1947, 1959, 1962), and Malinvaud's early Cowles papers and subsequent publications (1951, a, b, c, d; 1953; 1963, 1965a, b), and those of Koopmans (1957; 1959, 1963a, b, c, d, e; 1965, 1967) will also compared and analyzed in this regard, and Malinvaud's subsequent and *explicit criticism* of Koopmans's contribution to optimal growth (1972, 2003) will be presented and its meaning discussed, along with the previously unrecognized critique—based upon priority considerations—of Dreze (1964) ;

(ii) discussing "Koopmans's Mathematics" in the variant versions of his "optimal growth paper" over the period 1963-1965, and ;

assessing the "Cass-Malinvaud-Koopmans" nexus, that is to say, the importance of Cass's contribution, and Malinvaud's ongoing critiques of Koopmans's by means of the reconstruction of probable sequential cross-fertilization of the approaches to growth of Cass, Malinvaud, and Koopmans respectively.

I. Cass on "Optimum Saving", Ramsey, Optimum Growth and Pontryagin

Some thirty five years after what were, in effect, the first drafts (summer, 1963; Nov. 1963) of his thesis Chapter 1 (1965a), subsequently published in *RES* (1965b), an interview with David Cass appeared in *Macroeconomic Dynamics* (1998). Because of its importance for an understanding of the evolution of his work on optimum growth, *and the questions it raises in retrospect, that we will attempt to answer in this paper*, we analyze it in detail below.

In the interview, Cass recalled that he was led to his thesis topic on "optimal growth" by "the fascination Uzawa had with the [Pontryagin] maximum principle" (1998, 537). Cass went on to talk about "what we [Uzawa and Cass] wanted to do", that is to say "optimal or prescriptive growth", rather than "descriptive growth a la Solow". He continued on to relate how Uzawa proceeded, from his own early work on growth, to "optimal growth theory", with Uzawa himself, in effect, recreating "the calculus of variations", discovering "the maximum principle", and "becoming fascinated with it". Cass then recalled that "I was very influenced by Uzawa's work. I didn't even know about Ramsey at the time" (1998, 538). He was then asked "…But didn't some people know about Ramsey? Didn't Uzawa?" to which Cass frankly replied: "No. I don't think so, *because I didn't find out about Ramsey until after I had written the first chapter of my thesis on optimum growth*. And then, I was, to be perfectly honest, I was a bit embarrassed about it" [our emphasis] (1998, 538). Cass was then asked "How did you discover Ramsey?", and replied (1998, 538):

> I don't remember now. Maybe somebody mentioned it; maybe Uzawa knew about it, but not really, because he thought my contribution was absolutely seminal. In a way it is not at all. In fact I always have been kind of embarrassed because that paper is always cited although now I think of it as an exercise, almost re-creating and going a little beyond the Ramsey model.

Cass was then asked about the relationship between his thesis approach, the "Ramsey problem," transversality, and the difference between "discounting" [Cass] and "no-discounting" [Ramsey and Koopmans] cases. He replied (1998, 538-9):

> As a technical aside, it is very interesting that the Ramsey problem is a counterexample to something which people now always do. I think they do it in macro without even thinking about it, when they do dynamic optimization, and they write down transversality conditions as necessary, which I also said something about in my thesis, and this is dead wrong. The Ramsey problem is a counterexample to this: You have an optimum, but it doesn't satisfy the transversality condition.

When asked: "Is this an issue only in the no-discounting case? " , Cass responded (1998, 539):

> Yeah, that's in the undiscounted case. It has to do with the condition in capital theory that is called nontightness, which is a sufficient condition for the transversality condition to be necessary, and basically is an interiority condition that enables you to use a separating hyperplane theorem.

Cass was then asked, again, how he "came to the optimal growth problem". Due to its importance, his detailed response (1998, 539) is reproduced below in full:

> Actually, even though Uzawa always went back and read literature and was always motivated by literature, I didn't pick that up from him at all; I just decided to work on this problem because the techniques were new and exciting and it seemed like an interesting problem. So I taught myself the maximum principle, some differential equations, and so on, by talking to people, seeing Uzawa working, and basically reading math books. Our bible at the time was Pontryagin's original book [translated] on the maximum principle. That is really interesting too, because that book is very geometric, and Pontryagin's blind…and yet all of his thinking is purely geometric; he pictures things. So I just put the two together, and then Uzawa thought this was great. I'm not sure why, I guess probably because Tjalling Koopmans was working on this problem and Tjalling was a bit of an idol for Uzawa. Actually, Uzawa liked to one-up people. At some point he was talking to Tjalling about the problem, and Tjalling was describing what he was doing and Uzawa interrupted and said, "Well, I have a graduate student who did that problem." Then Tjalling got very nervous about it, he was always very nervous about . . ., oh, authorship and who was first and that sort of thing, and we had some correspondence. Koopmans was also very interested in the no-discounting case, so he solved the much harder problem in some ways, in addition to solving the problem with discounting. Tjalling did all his analysis from first principles; he derived all of the conditions.

Cass's 1998 retrospective interview raises the following questions:

(i) How did his thesis Chapter 1 (1965a) and *RES* paper (1965b) which was based upon it, and his treatment of Ramsey, evolve?

(ii) Given that, according to Cass's recollections, Uzawa seems to have played a role in possible cross-fertilization between Cass's thesis and Koopmans's work, how and when did this occur, and what ideas may have been the subject of exchange between them, via Uzawa?

Below, and in section 4 of the paper, we try to answer these, and related questions.

In June 1965, Cass's Stanford Ph.D thesis, entitled "Studies in the Theory of Optimum Economic Growth," supervised by Uzawa, was submitted and approved by his thesis committee, consisting of Uzawa, Maddala, and Amemiya (1965a, ii)**.** The first chapter was entitled "Optimum Growth in an Aggregative Model of Capital Accumulation;" the second "Optimum Growth in the Two-Sector Model of Capital Accumulation: an Extension;" the third, "Optimum Economic Growth in an Aggregative Model of Capital Accumulation: a Turnpike Theorem."

In the preface to his thesis, Cass wrote:

> The question of how best to allocate an economy's current resources between provision for the present and accumulation for the future, in short, the problem of optimum economic growth, is clearly an important one. Especially in the underdeveloped countries the problem constitutes a central issue in contemporary economic policy. Moreover, only by its rigorous analysis may we hope to provide standards by which to evaluate the long run performance of saving-investment institutions in the developed countries.

Cass continued:

> The purpose of the self-contained studies presented here is to investigate the problem of optimum economic growth within the abstract framework of closed, fixed technology, neoclassical growth models and under the simple criterion of maximizing the total welfare derived from consumption over time. Though important aspects of the growth process, as well as important considerations in judging it, are thus intentionally neglected, it is my firm belief that the studies represent necessary requisites for further understanding the problem.

He went on to acknowledge the "contribution" of Uzawa, and "stimulation" of his fellow students, Aaron Douglas, Steven Goldman, Harl Ryder and Karl Shell (1965a, iii).

The "self-contained" aspect of Cass's thesis chapters are manifest in the fact that even before its submission and approval, in June 1965, chapters appeared in the form of Stanford and Chicago Technical Reports respectively. In the former case, an early version of Chapter 1 ostensibly entitled "Optimum *Savings* in an Aggregative Model of Capital Accumulation" [our emphasis], as cited by Koopmans (1965, 286), was circulated, in November 1963, under the aegis of the Stanford IMSSS. We will discuss the importance of this title and document below. In the latter case, it would seem that most probably Chapters 1 and 3 were circulated as a University of Chicago Technical Report, under the title "Studies in the Theory of Optimum Economic Growth," identical to his thesis title, in the summer of 1964; but again, more about this below. At this point, then, let us turn to a detailed textual comparison of Chapter 1 of Cass's thesis and the version as published in his now classic 1965 *RES* paper.

Cass opened Chapter 1 of his thesis by writing:

> One of the crucial issues of economic policy in underdeveloped countries is the question of how much of a society's current output should be accumulated. The purpose of this paper is to investigate the optimum saving pattern with respect to one objective, the maximization of social welfare as measured by the total of the discounted utility of per capita consumption. Other, possibly conflicting, objectives are avoided and the complexity of any existing economy is simplified by assuming a central planning authority which directs production and allocates current output between consumption and investment, within the closed, aggregative model introduced by Solow [8] and Swan [10].

Interestingly enough, the first note to Chapter 1 stated that "very useful comments by a referee of RES are also gratefully acknowledged." Based upon this note, one could, at first, conclude that Ch. 1 of the thesis was simply reproduced as Cass's 1965 *RES* paper. Textual comparison of thesis and published versions,

however, tell a very different story, as there are many changes in, and elisions from the text between the two versions. Among these is the elision in the published version of the fact that Cass spent the summer of 1964, along with Uzawa, and fellow Stanford graduate students Shell, Ryder, Douglas, and Goldman, at the University of Chicago, where he put together the TR with the same title as his subsequent thesis; but more about this below. Other elisions include references, such as that to Swan, cited above. Indeed, the degree of amendment and elision attests to the possibility that the revision process between thesis and published version *may have actually occurred in two stages*, with Cass's initial focus on "optimum saving" almost completely elided at the request of the *RES* referee. This is manifest in the fact that while in the thesis version there is wide-ranging discussion of the issue of the "saving-investment ratio" and "optimum saving policy," consideration of optimum saving only appears in one section of the published version (1965b, 238, section 6), mention of underdeveloped countries elided almost entirely -- except for passing mention of a "capital-poor economy" (1965b, 238) .

On the other hand, population and population growth were added to the *RES* version of the model (1965b, 234). Other significant amendments and elisions included equations and notation relating to saving, the saving-investment ratio and optimum saving policy (1965a, 3,19) and a "calibration" table relating the social discount rate, growth rate of labor force and capital-labor ratio (1965 a, 24). And last, but not least, in note 2 of chapter 1 in the thesis, Cass wrote "After the original version of this paper was completed, a very similar analysis by Koopmans …came to our attention. We draw on his results in discussing this limiting case." (1965a, 2, note 2). In the *RES* version, he added the phrase "where the effective social discount rate goes to zero" (1965b, 233, note 2) .

The shift in emphasis away from "optimum saving" and the problem of capital in "underdeveloped countries" (1965a,1), can be seen in the change between the "central result" of the thesis and published versions of Chapter 1. In the former case, "the central result" was stated "as a theorem" yielding the "unique optimum path". This, in turn, encompassed, by definition the "unique optimum policy" based upon what was, in effect, the optimum saving-investment ratio (1965a, 20). In the latter case, the theorem summarizing the "central result" also described the "unique optimum path", albeit *without* reference to either the saving-investment ratio or the "unique optimum policy" related to it (1965b, 238).

With regard to the "stimulation afforded by working closely" with his "fellow students" (1965 a, preface, iii), in Chapter 1 of the thesis, Cass referred to a "fact… originally developed by H. Ryder," regarding "optimum policy." This was "that for a Cobb-Douglas production function and a logarithmic utility function, the optimum policy decreases (increases) steadily to s*, starting from k (0) < k* (k*< k (0)." (1965 a, 21-22, and 22 note 4). In the *RES* version, this passage does not appear, consistent with the elision of almost all reference to saving and the saving-investment ratio there, as noted above.

Now, one of the most problematic aspects of constructing a time-line for the evolution of Ch. 1 from thesis to published version is the fact that in the thesis itself—*in Chapters 2 and 3*—and in the published version of Koopmans's Vatican paper (1965), a Stanford Technical Report, written by Cass, was cited; in the former cases, entitled "Optimum *Saving* in an Aggregative Model of Capital Accumulation," albeit without being dated by Cass (1965a, 25, 89); in the latter case, entitled "Optimum *Savings* in a Aggregative Model of Capital Accumulation," *dated by Koopmans* as Nov. 27, 1963 (1965, 286). Mention of this Stanford TR, however, was elided in the

published version of Ch. 3 of Cass's thesis, that his, the *Econometrica* version of his seminal "Turnpike" paper (1966).

Now, given Cass's recollection (1998) that he had not been aware of Ramsey's work until after having written the first chapter of his thesis, and that thesis Chapter 1 as submitted in June 1965 (1965a) and published, in revised form, in *RES* in July 1965 (1965b), did utilize and cite Ramsey's approach, it would seem that the first drafts of Cass's thesis date from summer 1963, that is to say, his Chicago Summer Workshop Working Paper, and what Koopmans, and Cass himself, identified as the Stanford Technical Report, albeit only with Koopmans dating it to November 1963. These first drafts, then, would be those that did not contain any reference to Ramsey, on Cass's own account (1998). However, by 1965, Ramsey's approach plays a prominent part in Ch. 1 of Cass's thesis as submitted, and in his *RES* paper. The question remains, however: *who directed Cass towards Ramsey's work?* We will attempt to answer this in part 4 of the paper below.

II. Allais and Malinvaud: "capitalistic optimum", capital accumulation and optimal growth

In 1947, Allais published *Economie et intérét* which presented what he called "the theory of a capitalistic optimum" (1947, 179-228). This work influenced economists dealing with capital accumulation and growth such as Malinvaud (1951a, b, c, d; 1953) and Desrousseaux (1959, 1961 a, b), among others.

Over the period February-September 1951—while at the Cowles commission—Malinvaud wrote four papers on the subject of the optimum amount of capital and capital accumulation. His first Cowles discussion paper, dated 9 February 1951, was on "welfare economics" and " optimum amount of capital in a stationary economy"

(1951a). The second discussion paper, dated 10 April 1951, was entitled "Efficient Allocation of Resources and Capital Accumulation" (1951b). Given the title of *this* discussion paper, it could be ostensibly considered as the "first draft" of what was to become his 1953 *Econometrica* paper, "Capital Accumulation and Efficient Allocation of Resources." Malinvaud presented a revised version of this paper at the September 1951 Minneapolis meeting of the Econometric Society (4-7 September), entitled "Efficient Accumulation of Capital" (1953c) [Report of 1951 Meeting, 89]. Later the same month (17 September 1951), Malinvaud produced a Cowles discussion paper, numbered 2026, entitled "Capital Accumulation and Efficient Allocation of Resources" (1951d). In the first note to his seminal 1953 *Econometrica* paper, Malinvaud wrote that it was "based on" his "Cowles discussion paper No. 2026…and a paper presented at the Minneapolis a meeting of the Econometric Society in September 1951" (1953, 233, note 1). Now, this is not the place for detailed textual analysis of the variorum drafts of Malinvaud's 1953 *Econometrica* paper. Suffice it to say, however, that there are no *substantial* differences between his 1951 Cowles discussion paper, and his 1953 *Econometrica* paper of the same title -- especially the section entitled "Optimum amount of Capital." What is important to note here is the "emphasis" Malinvaud put on the contribution of Allais (1947) to the concept in his 1951 paper (1951d, 26,note 8), and his "influence" on the 1953 version (1953, 233,note 1). In his later interpretation of Malinvaud's 1953 paper, Koopmans *did not cite Allais* (Koopmans 1957, 105-25), despite the fact that he would have been aware of the work of Allais -- via his interaction with Malinvaud at Cowles.

On 25 May 1959, Allais himself gave a seminar at the Cowles Foundation, entitled "Influence of the Capital-Output Ratio on Real National Income" (Cowles Foundation Seminars, 1956-2004). This was essentially the first draft of his Bowley-

Walras Lecture delivered at the Econometric Society meeting on 28 December 1961, later published, in adapted form, under the same title as his Cowles seminar, in *Econometrica* in October 1962, although Allais did not cite his 1959 seminar paper in his later paper (1962, 700, note 1).

Allais noted the problematic aspect of Koopmans's not citing his work, as against Malinvaud's treatment of it, in his 1962 *Econometrica* paper "The Influence of the Capital-Output Ratio on Real National Income," when he counterpointed the work of Malinvaud and Koopmans regarding the "capitalistic optimum" as below (1962, 726):

MALINVAUD, EDMOND, 1953: "Capital Accumulation and Efficient Allocation of Resources"…
This article gives a new demonstration of the capitalistic optimum for i = 0, with reference to previous works.
KOOPMANS, T.C., 1957: The State of Economic Science, McGraw-Hill.
This book gives some indications, although very succinct, about the theory of a capitalistic optimum with only one reference to previous works: Malinvaud's article (1953).

And indeed, in the first part of his 1957 book, Koopmans devotes an entire section -- some twenty pages (1957, 105-126) -- to a detailed discussion of Malinvaud (1953).

In August 1961, Malinvaud, then at Berkeley, put out a Working Paper in the Management Science Research Center Series entitled "On the rate of interest of efficient proportional growth programs." He presented it at the Econometric Society meeting on 1 September 1961 in a session at which Uzawa also presented a paper extending his own two sector growth model (Report of 1961 Meeting, 373).

Malinvaud's "Vatican paper," which was presented at the October 1963 Vatican conference, was entitled "Croissances Optimales dans un Modele Macroecononomique," that is "Optimal Growth in a Macroeconomic Model." He noted that it was the subject of lectures he gave over the period May-June 1963 at the

ENSAE/EPHE conference (1965a, 309). It also appeared as an INSEE paper in 1963. Malinvaud's 1963 Vatican Paper was published, along with that of Koopmans, in the Vatican conference volume (1965a, 301-378). In his English "oral presentation," Malinvaud mentioned the similarities between his model and that of Koopmans, albeit his model being in discrete time, while that of Koopmans in continuous time. He concluded his English summary by saying that his approach "with discrete time is really quite similar to the one proposed by Ramsey for a continuous-time model," on the one hand, and to that of Koopmans, on the other (1965a, 308). We will deal with the discussion of Malinvaud's paper including the observations of Allais, the detailed comments of Koopmans, and Malinvaud's *crucial counter-example*, *as set out in his reply to Koopmans*, in section IV below.

In 1965, Malinvaud also published a paper entitled "Les Croissances Optimales" ["Optimal Growth"] in *Cahiers du Séminaire d' Économétrie* (1965b). This was presented in two parts. The first was entitled "Prologue to the study of optimal growth." Malinvaud cited Allais (1947, 1962), Koopmans's 1959 *Quarterly Journal of Economics* paper and 1963 Vatican paper, and his own 1953 *Econometrica* paper, and 1963 Vatican paper.

The second part was entitled "On the determination of optimal growth." Here he again cited his own 1963 Vatican presentation, but this time *only* Koopmans's 1959 *QJE* paper. It should be mentioned here that Malinvaud's citation of Koopmans's 1963 Vatican paper, in the first part of this article, *was only in the context of a note listing then recent work on growth* (1965b,77, note 3), including that of Kurz (1965), Srinivasan (1964), Stoleru (1965), and Uzawa (1963), among others. That Malinvaud did not cite Koopmans's 1963 Vatican paper in the *substantive* part of his own paper, dealing with what he called "the model" of optimal growth (1965b, 79) is, in our

view, the first indication that he did not consider Koopmans's 1963 [1965] contribution to be an important one, as will be seen below.

III. Koopmans contributions: from "maximal" and "proportional" to "optimal" growth

In July 1963, Koopmans presented a paper entitled "Economic growth at a maximal rate" at the Copenhagen meeting of the Econometric Society (1963b). The paper was given at the session "economic theory and method" chaired by Johansen, along with papers by Strotz and Theil. Prior to its presentation, Koopmans circulated a detailed "abstract" of what he called "an expository paper to be presented" at the abovementioned meeting. The paper itself was published in August 1964 in *QJE*. In an opening note, Koopmans wrote that "the ideas" for the paper "were developed" while he visited Harvard in 1960-61, and that "the paper was written after returning to Yale," the "research being undertaken" by Cowles, sponsored by ONR and NSF funding. However, in his 1964 *QJE* paper, the fact that he had presented the paper in July 1963 at the Copenhagen meeting of the Econometric Society was not mentioned by Koopmans. But, more interestingly, in the concluding paragraph of his *QJE* paper, Koopmans wrote:

> It should now be admitted that the problem of growth at a maximal rate is still a somewhat narrow and perhaps unnatural one. One would want to go on to the further study of optimal growth where the criterion of optimality expresses a concern with the desire for consumption levels that, if possible, are at all times above the minimum needed for self reproduction of an efficient labor force, at the no longer maximal but now "optimal" growth rate envisaged. One may want further to leave scope for an uncertain degree and kind of technological progress, and for a desire for flexibility in future consumer's preference.

In this context, Koopmans cited his forthcoming paper "On flexibility of future preference" (1964, 394 and note 4).

Now, a number of things deserve consideration here. First of all, in January 1963, Cowles Discussion Paper [DP] 152, entitled "Proportional growth and turnpike theorems," written by Koopmans (1963a) appeared. A comparison of DP 152, dated 24 January 1963, with the detailed abstract of the paper he presented (1963b), and especially with his August 1964 *QJE* paper, reveals that the *QJE* paper was actually a revised version of Cowles DP 152 that Koopmans had written over a year and a half earlier. Moreover, while the title was changed from "Proportional growth and turnpike theorems" to "Economic growth at a maximal rate," the final paragraph of Cowles DP 152 advocates going "on to the further study of optimal growth" and is identical to the final paragraph of Koopmans's 1964 *QJE* paper, but there is no mention of DP 152 in the *QJE* paper (1963a, 52-53; 1964, 39).

Second, in his Cowles DP 163, dated 6 December 1963 and entitled "On the concept of optimal economic growth" (1963d), Koopmans did not mention either his January 1963 DP 152 "Proportional growth and turnpike theorems" or the maximal growth paper he gave at the Copenhagen meeting in July 1963, but did cite his December 1962 Cowles DP 150, entitled " On flexibility of future preference", where he attempted "to express formally the idea of a present preference for flexibility in future preferences" (1962; 1963d, 34). Indeed, he had already cited this 1962 DP in his January 1963 Cowles DP 152 (1963a, 53). As this 1962 Cowles discussion paper is almost identical to the 1964 version (in Bryan and Shelley, 1964, 243-254), he cited in his 1964 *QJE* paper as forthcoming (1964, 394), it is somewhat surprising that Koopmans did not cite the 1962 version in his 1964 *QJE* paper in its stead.

This may seem a minor issue, but for the fact that in his review of the volume of Koopman's *Scientific Papers*, Malinvaud noted that these ideas actually had their origin in a paper entitled "Utility Analysis of Decisions Affecting Future Well-

Being", Koopmans had given at the Boulder meeting of the Econometric Society in 1949 (Report of 1949 Meeting, 174-5) that later appeared, in French, in 1953, and was eventually published in 1964. But Malinvaud also overlooked Koopmans and Bausch (1959) and Koopmans's 1962 Cowles DP 150, while citing the abstract of the 1949 paper "that appeared in *Econometrica* in April 1950" (Malinvaud, 1972, 799). Moreover, Koopmans himself did not cite his own Cowles DP 150 of December 1962, although he did cite the abstract of his 1949 paper in his 1964 version of "On flexibility of future preference" (1964, 254). Furthermore, Koopmans *himself* did not cite his *own* survey, with Bausch, of maximal and proportional growth (1959,111-117) .

Another "puzzle" emerges from a comparison of the title of the "forthcoming" paper given by Koopmans at the July 1963 Cambridge Conference, which will be discussed below. The title cited was "Proportional Growth and Turnpike Theorems" (Dreze, 1964, 60). The paper that actually appeared in the Conference volume (1967), was an amended version of Koopmans's 1964 *QJE* paper

A possible solution to these conundrums may lie in the fact that Koopmans also did not cite his December 1963 Cowles discussion paper on optimal economic growth in his 1964 *QJE* paper on "maximal" growth. Had he cited it, this would have upset the timeline of his narrative relating to the development of his ideas on both maximal and optimal growth.

Koopmans presented his Vatican, and Cowles paper, on optimal economic growth, at the Boston meeting of the Econometric Society on 29 Dec 1963, at the session on intertemporal economic theory, chaired by Strotz. Other presenters of papers were Dorfman and Radner. Interestingly enough Radner also presented a paper on optimal economic growth at the session. Discussants of the papers were

Chakravarty, Diamond, and Phelps (Report of 1963 Meeting, 702). The importance of this session will be dicussed in more detail in section IV below.

An additional puzzle arises from Koopman's Cowles DP 228, entitled "Intertemporal distribution and 'optimal' aggregate growth," that appeared in June 1967. According to Koopmans, "an earlier version was presented at a…meeting of the Econometric Society…held at Boston, December 1963" (1967, 1). However, the paper he actually gave at the Boston meeting on 29 December 1963, as noted above, was his Cowles DP 163 (dated 6 December 1963), and entitled "On the concept of optimal economic growth," according to the Report of the meeting published in Econometrica (Report of 1963 Meeting, 702). In his 1967 Cowles DP 228, Koopmans only refers to its published version (1965), and its reprint (1966); the Cowles DP 163 version is not cited at all (1967, 42). The significance of Koopman's citation practices regarding issues of priority and cross-fertilization of ideas will be seen below. Moreover, despite his ostensible movement towards the "problem" of "optimal growth", *as late as March 1965, Koopmans was still involved with the issue of "maximal proportional growth", this time, collaborating with Scarf, as evidenced in their Cowles Preliminary Discussion Paper entitled "Some suggestions for computing the von Neumann path of maximal proportional growth"*(Cowles Foundation, 1965).

IV. Conferences and Cross-Fertilization: the Malinvaud-Koopmans-Cass Connection and its Mathematics, 1963-1965

Over the period July-December 1963, three conferences took place -- the Cambridge conference in July, the Vatican conference in October, and the

econometric society conference in December -- at which Allais, Malinvaud, Koopmans, and Uzawa discussed the nature of capital formation and optimal growth.

The Cambridge Conference - July 1963

The papers presented, and discussion that took place at the IEA sponsored Cambridge Conference, appeared in 1967 in *Activity Analysis in the Theory of Growth and Planning*, edited by Malinvaud and Bacharach, but their impact on the evolution of *optimal* growth models has gone unrecognized up to now; even the recollections of McKenzie (1999), who attended the Cambridge Conference, and gave a paper entitled " Maximal Paths in the Von Neumann Model" (1967, 43-63), do not mention this. As noted above, the paper by Koopmans that appeared in the conference volume was an amended version of his 1964 *QJE* Paper, and thus did not deal with, or cite, Pontryagin's maximum principle. The conference was also attended by Allais and Malinvaud. Allais gave a paper entitled "Some analytical and practical aspects on the theory of capital"(1967, 64-110), while Malinvaud presented one entitled "Decentralized procedures for planning" (1967, 170-210). The discussion of Allais's paper related, among other things, to the impact of his own contributions -- such as his 1947 book -- the contributions of Malinvaud (1953, 1961), and that of Desrousseaux (1959, 1961a,b), and the relationship between them, with Allais stating outright that "the starting point of Malinvaud's thinking had been" his own 1947 book (1967, 295).

Now, a number of Conference papers, and the discussion of them, are relevant to our narrative, especially with regard to their stress on the importance and applicability of Pontryagin's maximum principle to the problems facing the theorist dealing with issues in capital theory and growth. Radner, in his conference paper "Dynamic programming of economic growth"(1967, 111-41), attempted to apply such methods

to "optimal growth" in an effort to "improve on" the application of "classical methods of calculus of variations," that had to date been largely confined to aggregate models like Ramsey's. In his discussion of the paper, Chakravarty pointed out that application of Pontryagain could be a better way to deal with the issue. Radner replied that "Pontryagin's technique certainly seemed a powerful one" (1967, 298).

Hurwicz , in his paper (1967, 142-149), dealt with "cases of current interest (in the context of Pontryagin's approach)" (1967, 144). In his paper, Chakravarty (1967, 150-169) explicitly applied "Pontryagin's maximum principle" to problems that were "extremely ackward to handle by the techniques of classical calculus of variations" (1967, 164-65). In the subsequent discussion of Hurwicz's paper, which had utilized Pontryagin's approach, as Hurwicz himself noted, according to the record, Koopmans *"said that he usually did not try to learn a new mathematical subject until he knew of a case for application"* (1967, 304) [our emphasis]. The importance of Koopmans' statement in the context of comments made by Chakravarty and Radner's response will be dealt with below.

The Vatican Conference - October 1963

Since the publication Koopmans' paper in the Vatican Conference volume (1965), its appearance as a Cowles Reprint (Cowles Foundation Paper 238), its "reissue" in the volume *The Econometric Approach to Development Planning* (1966), and its re-publication in Koopmans's *Scientific Papers* (1970), the discussion of his conference paper has been taken to be that which appeared *immediately* after the paper itself in the conference volume (1965, 289-300). A close examination of the conference volume, however, reveals that discussion of Koopman's original 1963 Vatican paper as presented, also took place after Malinvaud gave his paper. Indeed, Koopmans's original paper was further commented on in this context, by Haavelmo,

Allais, and Malinvaud, respectively, with a considered reply by Koopmans (1965, 380-84). This is not the place for detailed analysis of the further discussion of Koopman's original 1963 paper. Suffice it to say that Allais once again took the opportunity to take issue with both Koopmans's citation procedure and treatment of Allais's priority. Allais said (1965, 382):

> … My Econometrica Paper is only one study carrying forward things described in many preceding papers and I believe that I gave very precise consideration to the problem of the optimal path as long ago as 1947 in my book…that is fifteen years before that the Desrousseaux, Phelps, Joan Robinson, Swan and Von Weizsaecker studies with Professor Koopmans mentioned.

As mentioned above, Malinvaud gave a paper at the Vatican Conference which, when translated, is entitled, "Optimal Growth in a Macroeconomic Model". In reply to Koopmans's comments on his paper, *Malinvaud provided a "counter-example" which presented a fundamental problem that Koopmans was unable to resolve, until he applied Pontryagin's maximum,* as will be seen below. Because of its importance, and due to the fact that *no attention has been given to the cross-fertilization of ideas between Malinvaud and Koopmans at the Vatican Conference until now,* the verbal exchange between Malinvaud and Koopmans, regarding Malinvaud's "counter-example," and the mathematics underlying Koopman's original October 1963 Vatican paper (*as given in CFDP 163*), against that which appears in the 1965 Conference Volume, are given below. Let us first deal with the verbal exchange.

The model presented by Malinvaud was similar to that of Koopmans, *but focused on discrete, rather than continuous time* (1965a, 301-1), that is, a discrete time analog of the model examined by Koopmans, and as will be seen below, Cass. *In his paper, Malinvaud points up explicitly the need for a transversality condition, which in the paper he calls condition* "I" (1965a, 303, 322). In the discussion

following Malinvaud's paper, Koopmans asks Malinvaud whether or not it is the case that any trajectory not meeting Malinvaud's condition I "necessarily violates at some finite time, the sign restrictions on capital or consumption?"(1965a,379). Malinvaud replies that this is not the case, and cites the example with log utility and Cobb-Douglas production functions where there can be a "whole family of paths which meets the recursive requirements [Euler equations] and the sign restrictions" but are not optimal (1965a,380). At this point, Koopmans seems to drop out of the discussion until the end. Let us now turn to the issue of Koopmans's original Vatican Conference paper, as manifest in his CFDP 163, as against the 1965 published version.

Koopmans' Mathematics, 1963 vs. 1965

From Koopman's Vatican published version of the paper, the Euler equations are

$$\dot{z}_t = g(z_t) - x_t$$
$$\dot{x}_t = -\frac{u'(x_t)}{u''(x_t)}[g'(z_t) - \rho]$$

where $g(z) = f(z) - \lambda z$ and f is a standard production function in intensive form.

Now, let $\bar{z}$ be such that g($\bar{z}$)=0. Consider the allocation which has $z_t = \bar{z}$ and $x_t = 0$. From the Euler equations, we clearly have $\dot{z} = 0$. If we can show that at this allocation, we also have $\dot{x} = 0$, then this allocation will be a steady-state for the dynamic system defined by the Euler equations, and any trajectory on the stable manifold for this steady-state will necessarily converge to it only asymptotically, contradicting Koopmans' CDF163 assertion that convergence to the boundary along any non-optimal steady-state would happen in finite time.

Let us consider a CRR specification of the utility function, so that

$$u(x)=\left(\frac{x^{1-\alpha}}{1-\alpha}\right).$$

For this utility function,

$$u'(x)=x^{-\alpha}$$
$$u''(x)=-\alpha x^{-\alpha-1}.$$

For this specification of preferences, then

$$\frac{u'(x_t)}{u''(x_t)}=-\frac{x^{-\alpha}}{\alpha x^{-\alpha-1}}=-\frac{x}{\alpha}.$$

Since the production function *f* is strictly concave, $g'(\bar{z})-\rho=f'(\bar{z})-\lambda-\rho$ is finite. Hence, as $z_t \to \bar{z}$ and $x_t \to 0$, it follows that $\dot{x}_t \to 0$, and hence $(\bar{z},0)$ is a non-optimal steady-state solution for the Euler equations. (Note that the CRR utility functions embed the case of log utility when α=1, which is the example Malinvaud cites.) The asymptotic convergence along the stable manifold for this steady-state is the reason the transversality condition must be imposed in order to obtain the optimal solution.

How can we interpret the above? By comparing the earlier Cowles Discussion Paper 163 (1963), to the published version of the paper in the Vatican volume (1965), we found some major differences. The central result in Koopmans' 1963 Discussion Paper can be found on page 25, that is, proposition H. This proposition states that the Euler equations for the continuous time model with discounting are necessary and sufficient to characterize the optimal capital accumulation trajectory. We know this is wrong. Koopmans claims that the conditions are necessary and sufficient, because any path which is not optimal will violate the non-negativity restrictions on capital

and consumption *in finite time*. This is wrong because in his model, there is a steady-state equilibrium where all output is used to produce capital, and consumption is zero, as we have seen.

The proposition that corresponds to this in the published version of the paper is Proposition I on p.248 of the 1965 Vatican volume. Here, Koopmans has revised the proposition from the discussion paper to add the condition that in addition to the Euler equations (condition beta), the optimal trajectory must converge to the optimal steady-state equilibrium of the model (condition alpha). This condition is basically the central result of Cass's thesis chapter 1, except that Cass derived it from the fact that the Euler equations, together with the transversality condition, constitute the necessary and sufficient conditions for this kind of optimal control problem. A key question, then, is why didn't Koopman's simply adopt the transversality condition?

Given the above, it seems almost certain that Koopmans was able to convince himself that Malinvaud's counter-example to the claim that non-optimal paths violated the sign restrictions in finite time was valid not only in a discrete time model, but in the continuous time model as well. That he ends up citing Cass's November 1963 Technical Report in the 1965 version of the paper (1965, 286) also suggests that Cass's development and solution to the control problem via the Maximum Principle was the correct way to do it, so that Koopmans would need to include something akin to the transversality condition. While this is only conjecture (and of course, conjecture is all it can be), we suggest that Koopmans simply adopted Cass's condition that any optimal trajectory converge to the optimal steady-state as a way of avoiding being too obvious about the fact that he hadn't included the transversality condition in the paper he presented during the Vatican study week in October 1963. The question remains, however, who was it that pointed Koopman's to Cass's work?

In order to answer this, we now turn to the third of the crucial meetings in 1963, the Econometric Society Conference held in Boston, 27-29 December.

Econometric Society Conference - December 1963

The importance of Econometric Society Conferences, and the *cross-fertilization* that takes place at them has been documented in a number of cases, such as that of the September 1936 meeting, for which Hicks, Harrod, *and* Meade developed and presented the *IS-LM model*, and the December 1959 meeting, for which Muth *and* Mills developed and presented *Rational and Implicit Expectations* (Young, 1987; Darity and Young, 2001). The December 1963 Boston meeting of the Econometric Society can also be considered to be the meeting at which cross-fertilization in development of the optimal growth model took place. Indeed, a survey of the sessions and participants at the meeting shows that it was not only a *reprise* of the July 1963 Cambridge Conference, including almost all those who attended and presented papers -- Koopmans, McKenzie, Radner, Hurwicz, Chakravarty; with the exception of Allais and Malinvaud -- but also included a crucial *new* agent in the exchange of ideas, in the person of Uzawa. Closer examination of the Conference Report (Report of 1963 Meeting, 688-707) reveals even more regarding the interactions between these personalities in the sessions themselves.

Now, as noted above, Koopmans gave his Vatican paper "On the Concept of Optimal Economic Growth" at the meeting. The Conference Report stated that it was "to appear in a forthcoming volume published by the Pontifical Academy of Sciences" (Report of 1963 Meeting, 689). As only the title of his paper was listed in the Report of the Boston meeting, the question arises: *why did Koopmans actually put out two Cowles documents on optimal growth in December 1963*: his Cowles Foundation Discussion Paper 163, dated 6 December (1963d), and Cowles Foundation Discussion

Paper 163-A, dated 19 December, that is, the Appendix to CFDP 163 (Koopmans, 1963e) Moreover, there is the *conundrum* of the footnote in the CFDP 163-A, where Koopmans writes: "Readers of this appendix are requested to substitute two pages, numbered 23,24 found at the end of the appendix, for the pages bearing the same number in the article, CFDP 163, of which this is the appendix." (1963e, 55 note 1) This footnote appears to have been inserted just before the incorrect proof that the Euler equations alone are necessary and sufficient to characterize optimality. *But there are no pages at the end of the appendix number 23 and 24.* This would seem to suggest that Koopmans had found an error in the main draft, but the fact that he doesn't include any corrections is quite odd. That he would have wanted to include some corrections seems obvious, given that both the DP 163 and its Appendix are dated in December, which was *after* the Vatican conference. But the CFDP Appendix has the same incorrect assertion about violating non-negativity constraints in finite time. Our conjecture is that the Appendix was actually created *prior* to the Vatican conference, and then re-dated when Koopmans submitted it for the Econometric Society meeting. After the Vatican conference, he inserted the footnote pointing to the corrections, but for whatever reason, didn't include the corrections. He may have gone into the Econometric Society meetings hoping that in a short presentation no one would question the assertion that the Euler equations alone are necessary and sufficient.

But more is involved here than the conundrum of the Appendix to Koopmans' optimal growth paper. Indeed, there were many opportunities for productive exchanges of ideas and *cross-fertilization* that occurred at the Econometric Society Conference. These involve the two sessions on "economic growth" listed in the Report on the meeting, and the session at which the papers on "optimal growth" of

Koopmans and Radner, respectively, were presented, in addition to the session on "Topics in Economic Theory" (Report of 1963 Meeting, 690-707).

The first session on economic growth was held on Friday morning, December 27. Sidney G. Winter, University of California, Berkeley, was the Chairman. Papers were discussed by Michael D. Intriligator, University of California, Los Angeles; H. J. A. Kreyberg, University of Minnesota; and T. N. Srinivasan, Yale University. The papers presented were: "On an Asymptotic Non-Substitution Theorem in the Two-Sector Closed Production Model," by Emmanuel Drandakis, Yale University; "Maximal Paths in the von Neumann Model," by Lionel McKenzie, University of Rochester; and "Optimal Policy for Economic Growth," by L. Stoleru, France. The second session of economic growth was held on Sunday afternoon, December 29. Phoebus Dhrymes, Harvard University, was Chairman. Papers were discussed by John Fei, Yale University; Benton F. Massell, RAND ; and Hirofumi Uzawa, Stanford University. The papers presented were: "Neoclassical Growth and the Efficient Program of Capital Accumulation," by Hiroshi Atsumi, Osaka University, and University of Rochester; "Some Implications of Diminishing Returns in Economic Growth," by Martin J. Beckmann, University of Bonn, West Germany, and Brown University; and "On the Stability of Growth Equilibrium," by Ryizo Sato, of the University of Hawaii (Report of 1963 Meeting, 690-91; 706-7)

In between these two sessions on growth, a session on what was called "Intertemporal Economic Theory" took place, on Sunday morning, December 29. Robert H. Strotz, Northwestern University, was Chairman. Papers were discussed by Sukhamoy Chakravarty, New Delhi School of Economics, India, and Massachusetts Institute of Technology; Peter A. Diamond, University of California, Berkeley; and Edmund S. Phelps, Yale University. The first paper presented was by Robert

Dorfman, of Harvard, entitled "Irving Fisher and the Theory of Corporate Investment." Next, Koopmans gave his Vatican paper, "On the Concept of Optimal Economic Growth and its Appendix. Finally, Roy Radner, then at the University of California, Berkeley, gave his paper entitled "Optimal Economic Growth" (Report of 1963 Meeting, 702).

On the same day that the first session on growth was held, Uzawa also gave a paper with Hurwicz, albeit on Friday afternoon, 27 December, at the session "Topics in Economic Theory," chaired by George Stigler, of Chicago. Papers were discussed by Gary S. Becker, Columbia University and NBER; Joseph McGuire, University of Kansas; and Menahem E. Yaari, Yale University. The papers that were presented were: "The Evaluation of Infinite Utility Streams," by Peter Diamond, of the University of California, Berkeley; "On the Integrability of Demand Functions," by Hurwicz and Uzawa; and Sidney Winter, "Some Differential Equation Models of Economic 'Natural' Selection" (Report of 1963 Meeting, 693-94).

Given the extent of the possible interactions of the individuals listed above, and their mutual interests, we can conjecture *the following possible paths of sequential cross-fertilization***.** With regard to the participants in the growth sessions**,** Srinivasan, McKenzie, and Uzawa would most probably have discussed various aspects of their work. At the session "Intertemporal Economic Theory", it is likely that Chakravarty, Phelps, Radner, and Koopmans discussed issues relating to optimal growth, *but not necessarily the problem faced by Koopmans, as raised by Malinvaud at the Vatican Conference*. Hurwicz and Uzawa were already familiar with Pontryagin's approach, but would not have discussed its application to problems in growth theory, given the nature of their own joint paper. But what of a possible exchange between Koopmans and Uzawa ?

V. Independent and Simultaneous Discovery vs. Sequential Cross-Fertilization: Pontryagin, Ramsey, Originality and Transversality

Taking Cass's recollection regarding the Koopmans-Uzawa exchange, as related to him by Uzawa, then, *we would assert that this exchange took place at the Boston meeting of the Econometric Society, either before, or after, Koopmans' presentation of his original 1963 paper, which was his Vatican paper.* But more is involved here than Uzawa simply doing a "one-up" on his "idol," Koopmans, by mentioning the "work of his graduate student," according to Cass. We conjecture that either at, or sometime after the Boston meeting, *Uzawa gave Koopmans a copy of the November 1963 draft of Cass's thesis Chapter 1 ("Stanford Technical Report 5"), which Koopmans then cited in the 1965 version of his Vatican paper*. After seeing this, *Koopmans probably recognized the efficacy of applying Pontryagin to solve the problem raised by Malinvaud's counter-example at the Vatican Conference,* following from his own maxim that "*he usually did not try to learn a new mathematical subject until he knew of a case for application"* (1967, 304) [once again, our emphasis]. Now, as Cass's result derived from the primitive of the first-order conditions plus transversality, which embodied Malinvaud's theorem, Koopmans simply decided to use Cass's result, since the combination of the Euler equations plus convergence to the optimal steady-state were sufficient to characterize the optimal growth path. Koopmans then cited Cass's November 1963 "Stanford Technical Report," that is, the first draft of Cass's thesis Chapter 1, in the 1965 version of his Vatican paper, though he did not mention that this wasn't just a passing reference, *but rather a key correction of the results from his original 1963 Vatican paper.*

Moreover, after Koopmans had seen the November 1963 draft of Cass's thesis Chapter 1, *he could very well have pointed out to Uzawa that Cass had not dealt with the work of Ramsey, or even cited Ramsey's contribution.* On his own account, Cass didn't find out about Ramsey until after he *"had written the first chapter"* of his thesis (1998, 538). We think that it was most probably Koopmans who set Uzawa and Cass on to Ramsey's continuous time model, thus completing the sequential cross-fertilization that contributed to the 1965 papers of Cass and Koopmans respectively.

But again, more may be involved here than Koopmans' deference to Ramsey (1965d, 2-4). Close examination of Cass's thesis Chapter 1 indicates that it could have been, and was indeed developed, without Ramsey, as it followed the basic structure of Solow-Swan. Koopmans, however, *while not necessarily citing all those who may have influenced his own work, did have a propensity to ask others to cite previously published works for "diplomatic and priority reasons".* This is evident from a letter from Koopmans to Herbert Simon dated 19 June 1951 regarding the draft of what was to become Simon's classic paper "Causal Ordering and Identifiability" in Hood and Koopmans *Studies in Econometric Method* (1953). In this, Koopmans told Simon

> Yesterday, I got to see a brief review by Herman Wold…, concentrating on certain comparative aspects with his work. It appears from that, which I had realized before, but somehow forgotten, that they make much of the general order of variables brought about by putting the matrix β in triangular form and the matrix Z (co-variances of disturbances in diagonal form) simultaneously. I wonder if, for diplomatic and priority reasons, you could make some bows and references in their direction in the causal ordering article.

Koopmans went on to list suggested references to the work of Wold and his "group" for possible inclusion in the Simon paper; and Simon included two of them in his paper accordingly.

In any event, there still remains the issue of the *originality* of Koopmans' approach when counterpointed to Malinvaud's contribution, and that relating transversality, in

the story we have been trying to make sense of here. With regard to the first issue, as early as 1964 Dreze noted (1964, 36, note 128):

> Interest in optimal capital endowments has more recently been revived in the United States, for instance, in connection with the "golden rule of accumulation" introduced by Phelps…and with the work on turnpike theorems and optimal economic growth -- see e.g., Koopmans [1957]. No attempt will be made here to integrate the two bodies of literature. The informed reader will undoubtedly notice some striking similarities as well as the definite originality of the French contributions [of Allais (1947) and Malinvaud (1953)].

The second issue regarding the Koopmans-Malinvaud nexus flows from Malinvaud's *JEL* review of Koopmans' *Scientific Papers*. He wrote (1972, 799):

> The reader will like his lucid presentation of turnpike theory in "Economic Growth at a Maximal Rate," and of the optimal growth model in "On the Concept of Optimal Economic Growth." *But the most significant contribution of Koopmans in this field was to have considerably clarified the concepts and puzzling problems that are raised by the search for optimality over time. From this point of view once should particularly recommend "Objective Constraints and Outcomes in Optimal Growth Models," his Irving Fisher lecture at the Warsaw meeting of the Econometric Society in 1966* [our emphasis]**.**

Malinvaud's assessment goes against the conventional wisdom regarding the Koopmans paper , which puts it on par with that of Ramsey and Cass and more significant than Malinvaud's own contributions (1953; 1965). With regard to "transversality"conditions , however, Malinvaud's assessment may be valid. Indeed, in a 2003 interview with Malinvaud published in the *Journal of Economic Perspectives,* Malinvaud replied to a question put by Alan Krueger regarding the impact of his own work on capital accumulation (2003, 188-89), by saying that "*this work contributed to make mathematical economists understand why they should pay attention to transversality conditions"* [our emphasis]; something which, as shown in this paper, Koopmans did not do, until pointed in that direction by Malinvaud, on the one hand, and by Cass, on the other.

Conclusion

Priority in scientific discovery -- together with the dictum not to reinvent the wheel -- is at the heart of the incentive structure that motivates scientists in their research. At the same time, openness of communication and the rapid sharing of ideas are critical for the progress of science. In our examination of the Cass-Malinvaud-Koopmans nexus, we see a clear example of the interplay of these two forces in the development of the neo-classical model of economic growth. Indeed, the scenario we presented here is probably as old as science itself. On the one hand, we see a serious and motivated young graduate student in Dave Cass, striking out to solve a problem he probably thought no one else had thought of or worked on, using the newly developed mathematical techniques pioneered by Pontryagin and his students. In Tjalling Koopmans we find an established and revered senior scientist also tackling the same problem with a clear understanding of its import and history, but lacking the up-to-date mathematical tools needed to fully solve the problem. In Edmond Malinvaud, we have a true pioneer who formulated and solved a version of the discrete-time growth model without the aid of Pontryagin's mathematics, but whose practice of publishing much of his work in his native French limited the exposure of his work.

In terms of the cross-fertilization that resulted from the communications we have outlined here, Cass clearly benefitted from Koopmans' knowledge of the history of his subject and the pioneering role that Frank Ramsey played in developing the early growth model. Koopmans, in turn, benefitted from Cass's grasp of the new mathematics; indeed, Cass's work showed Koopmans explicitly that there was an application for the new mathematics of Pontryagin. Malinvaud's understanding of the

importance of transversality conditions for infinite-horizon growth models allowed him to identify Koopmans' error in the initial draft of his Vatican paper.

It is difficult to say whether the issue of priority in these studies was a matter of contention between the three players or not. From the Cass interview, we know that he was himself willing to accept the established view that his work and Koopmans' was done independently and simultaneously. Indeed, Cass's first job out of graduate school was at the Cowles foundation, and throughout his life, he professed nothing but the highest admiration for Tjalling Koopmans. Even so, his comments from the interview indicate that Koopmans did have a real concern for priority issues, something also confirmed from his letter to Herb Simon and reflected (we believe) in the change in his citation practices following this period. As for the relationship between Koopmans and Malinvaud, there seem to be undertones of criticism in Malinvaud's writing about Koopmans' contribution to growth theory, and while we tried to contact Malinvaud about this, the state of his health at the time of this writing was such that he could not reply.

While these kinds of professional jealousies are the price we pay to motivate research, they would never arise without the openness of the research process, and the kind of cross-fertilization of ideas this openness permits. The sequential communication between Cass, Koopmans and Malinvaud, mediated by the conference process, resulted in a model that put growth on a fully general equilibrium footing. In the process, it revolutionized the study of growth and set a standard for dynamic economic analysis that has been followed ever since.

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